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IX. *Hue Difference and Flicker Photometer Speed.*

By HERBERT E. IVES *.

IN the first of the writer's papers on the flicker photometer some data were given on the speeds of operation of the instrument when the luminosity of the spectrum was measured against a carbon-lamp comparison standard. These speeds, which are *critical speeds* for the position of intensity match, show a minimum near 58μ . In explanation of this minimum it was remarked †: "In order to compare lights of different colours it is necessary to attain such a speed that the colour flicker, *due to difference in hue*, disappears. It is therefore to be expected that at the ends of the spectrum where the hue is most different *from the comparison lamp*, a higher speed is necessary."

This explanation appeared adequate to the writer, and partly for this reason, partly because no quantitative theory was then available whose verification depended on fuller data, and partly because no flicker photometer then existed which was entirely free from purely mechanical flicker, or abrupt transitions which might in part behave as such, no further experiments were made on this line. Recently, however, Mr. L. T. Troland ‡ has published somewhat fuller data of the same kind, which he explains in a different manner. According to his view the wave-length-speed curve may be interpreted as the *reciprocal of the luminosity curve*, the minimum in the yellow-green indicating the greatest *whiteness*, which he considers as depending on the same underlying process as luminosity.

This view is so antagonistic to the present writer's ideas on the meaning of luminosity, and on the mechanism of intermittent vision as developed in recent theoretical papers §, that it appeared desirable to secure some additional experimental data, using the new polarization flicker photometer ||. These data, which are given below, appear to substantiate the theory upon which the experiments were based.

* Communicated by the Author.

† "Photometry of Lights of Different Colours," Ives, Phil. Mag. July 1912, p. 167. The italicizing is added in the quotation.

‡ "Apparent Brightness, its Conditions and Properties," Troland, Illuminating Engineering Society Convention, Sept. 1916.

§ "Theory of the Flicker Photometer," Ives & Kingsbury, Phil. Mag. Nov. 1914, p. 708, and April 1916, p. 290.

|| "A Polarization Flicker Photometer, and some Data of Theoretical Bearing obtained with it," Ives, Phil. Mag. Apr. 1917, p. 360.

Theory.

The greater part of the special theory necessary to handle this question is contained in the next preceding paper, on "A Polarization Flicker Photometer, &c."*, in the discussion of the brightness and hue discrimination fractions. It is there shown (equation 14), that the critical speed at the equality setting of two different colours is given by :

$$\omega_M = \frac{\omega_R + \omega_G}{2} \left(\frac{\log \frac{2}{\delta_H}}{\log \frac{2}{\delta_B}} \right)^2, \dots \dots \dots (1)$$

where ω_M is the critical speed of the mixture, ω_R and ω_G are the critical speeds for the two colours (R and G) separately, δ_B is the brightness discrimination fraction, and δ_H the hue discrimination fraction. The latter is defined as the difference in the quantity of one of the colours in the mixtures at the opposite phases, divided by the mean quantity, and it was pointed out that this fraction, unlike the brightness discrimination fraction, varies with the size of the colour difference.

For the purposes of the present paper it is convenient to consider this fraction in a slightly different light. Thus, instead of identifying it with one colour of the mixture only, it may be identified with both by considering it to represent a just distinguishable distance along a line of a colour-mixture diagram, divided by (half) the length of the line. It is thus twice the magnitude of the just distinguishable fraction that would be most naturally derived if the definition were developed solely from colour-mixture considerations. Now this just distinguishable distance along the colour-mixture line remains fixed, no matter how far in either direction the line is extended, but the value of the fraction decreases directly as the length of the line. Consequently, if we wish to learn the effect of increasing the colour difference between the lights compared (confining ourselves for the present to lights whose equal luminosity mixture is always the same), it is only necessary to consider the value of δ_H as varying inversely as this difference.

If we call the distance apart of the two compared colours

* Ives, *loc. cit.*

on a colour-mixture line c , in any convenient units, we then have the speed given by

$$\omega_M = \frac{\omega_R + \omega_G}{2} \left(\frac{\log \frac{2c}{\delta_H}}{\log \frac{2}{\delta_B}} \right)^2, \quad \dots \quad (2)$$

which may be simplified by the combination of constants to

$$\omega = M \left(\log \frac{2c}{\delta_H} \right)^2, \quad \dots \quad (3)$$

where M is a function of the working intensity. From inspection of this equation it is seen that the speed becomes zero when the two compared colours are separated on the mixture diagram by only the just distinguishable difference, and that the speeds go up apparently without limit as the colour difference increases. Actually, as experiment shows, no hues exist sufficiently far apart to make the critical speed of an equal luminosity mixture ever more than a fraction of the speed necessary to eliminate flicker of the coloured light against darkness.

In order to plot this equation on such a scale as to represent an actual case it is necessary to know the value of δ_H . One method of obtaining this was developed in the preceding paper. Another, bearing more directly on the present problem, may be outlined. Suppose two coloured lights of equal intensity to illuminate the two glasses of the mixture photometer described in the previous paper. Let these be represented as the end points of a straight line of convenient length c . Suppose the critical speed determined. Then let the two glasses be turned until each is illuminated by two parts of one light and one part of the other. The colour of each is then represented by a point one-third along the line from the end, and the colour-mixture distance of the two glasses is $\frac{1}{3}c$. Suppose the critical speed determined for this condition. We then have two equations from which δ_H can be found.

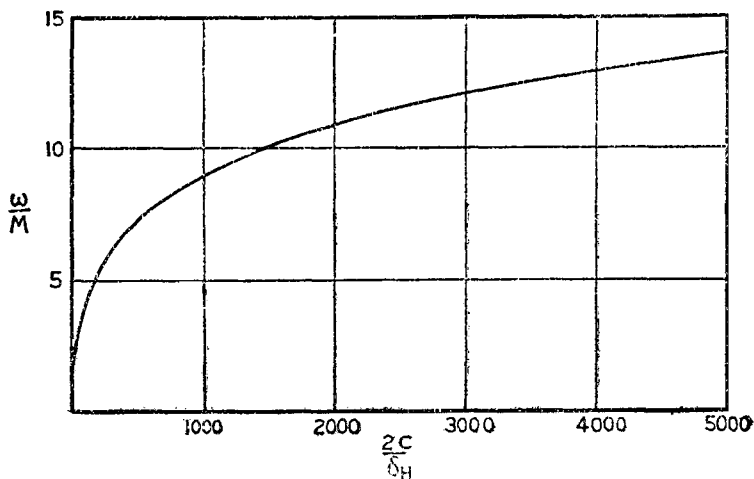
In fig. 1 is shown a plot of equation (3), in terms of $\frac{\omega}{M}$ against $\frac{2c}{\delta_H}$. Direct experimental verification of this

curve was not undertaken, since the curve shown by Troland * for mixtures of red and white light is closely of

* "Apparent Brightness, &c.," Illum. Eng. Soc. Convention, Sept. 1916.

this type, the deviations being no greater than can be explained by the fact that he moved the hue of the mixture continuously from one end of the mixture line to the other, thus probably encountering differences in the value of the just noticeable distance along this line.

Fig. 1.



Relationship between speed $\left(\frac{w}{M}\right)$ and hue difference $\left(\frac{2c}{H}\right)$, as calculated from theory.

In order to apply the theoretical work just outlined to the spectrum, it is necessary to know the position of the spectrum and any comparison light used in the complete colour-mixture diagram, namely, the colour triangle, and also the distances corresponding to equal hue steps in the triangle. It is important to note, moreover, that the colour triangle to be used is the equal luminosity one, and not the equal sensation-sum triangle usually plotted.

Assuming that we have such an equal luminosity triangle available with the equal hue intervals for all parts and all directions determined, the process to be gone through may be outlined as follows:—First, the distance between the spectrum colour and the comparison light is measured; the mid-point of the line joining them, that is, where the equal luminosity mixture occurs, is then found; the length of a just distinguishable hue difference at this point is read off, and the ratio of this length to half the whole distance gives

a fraction proportional to δ_H (assuming that successive presentation hue discrimination is proportional to juxtaposed presentation discrimination and proportional in the same way for different colours).

Suppose this calculation gone through for the whole spectrum; there can then be plotted a series of values of δ_H against wave-length, the value of the δ_H units being undetermined. If then the exact value of δ_H be found by the method above outlined, for any one wave-length, its value for other wave-lengths is at once obtainable from the plot just assumed made. The value of the constant into which the other terms of (1) can be combined (M) is obtained from the same observations, so that all the factors necessary to furnish the critical speeds have been found.

When it comes to carrying through this indicated process it is found that existing data are not adequate to an exact solution, and that the work to put what we have into the shape required is rather great. In order to translate the distances on the ordinary equal sensation-sum triangle, which are proportional to

$$\sqrt{(R_2 - R_1)^2 + (G_2 - G_1)^2 + (B_2 - B_1)^2}, \dots \quad (4)$$

where R, G, and B are the three sensation coefficients *, to distances on the equal luminosity triangle, it is necessary to know the luminosity values of the three sensations. If we call these L_R , L_G , and L_B , then the distances on the equal luminosity triangle are proportional to

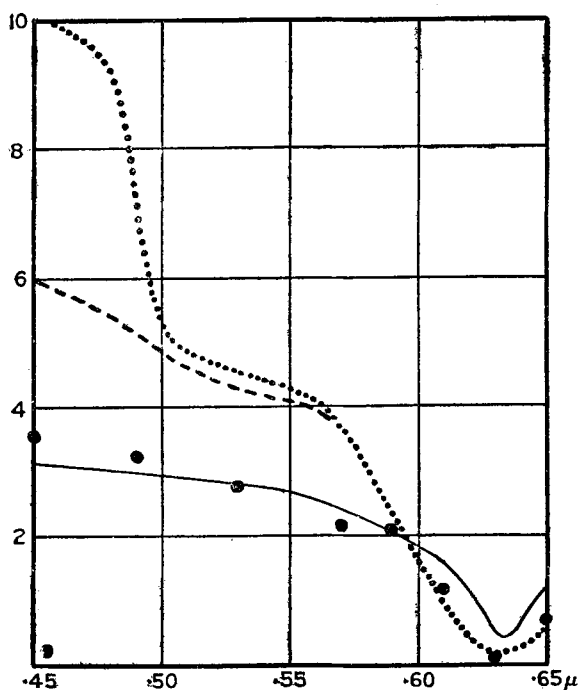
$$\sqrt{\left(\frac{R_2 - R_1}{L_R}\right)^2 + \left(\frac{G_2 - G_1}{L_G}\right)^2 + \left(\frac{B_2 - B_1}{L_B}\right)^2} \dots \quad (5)$$

I have carried through the calculation of the distance of white (centre of the ordinary triangle) from the various parts of the spectrum, using a modified triangle obtained from Koenig's colour sensation data, which has been recently correlated with luminosity, only to find that for all practical purposes the result is the same as though the distances had been measured in the ordinary triangle, the mixture point being given by the point distant from the two colours inversely as their luminosities, as given by (5). This fact, which is principally due to the enormous exaggeration of

* A discussion of colour-mixing diagrams, upon which the following is based, is given by the writer in the *Journal of the Franklin Institute*, Dec. 1915, p. 673: "The Transformation of Colour Mixture Equations from One System to Another."

the blue component necessary to bring it up to equal luminosity with the others, so that the third member of (5) practically has the whole say in any distance determination, simplifies the evaluation of colour differences in hue difference units, since such hue discrimination data as we have can be most easily grasped when plotted in the equal sensation triangle.

In figs. 2 *a*, *b*, *c*, and *d* are shown (dotted lines) the distances of the various parts of the spectrum respectively

Fig. 2 *a*.

Dotted lines: Colour Triangle distances.

Dashed lines: Corrected distances, taking into account variable size of equal hue distances at equal luminosity mixture points.

Full lines: Speeds as calculated for dashed line data and relationship plotted in fig. 1.

Dots: Experimentally found speeds, in arbitrary units.

2 *a*.—Spectrum against red light.

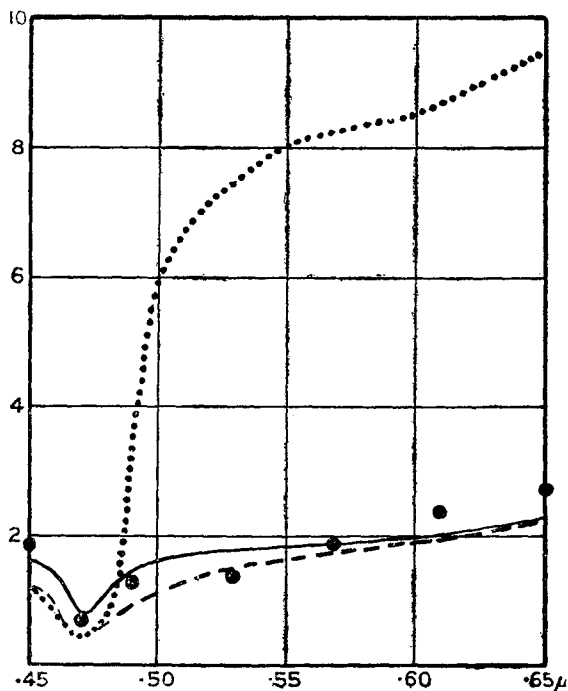
2 *b*.—Spectrum against blue light.

2 *c*.—Spectrum against yellow-white (tungsten lamp).

2 *d*.—Spectrum against white (5000° black body).

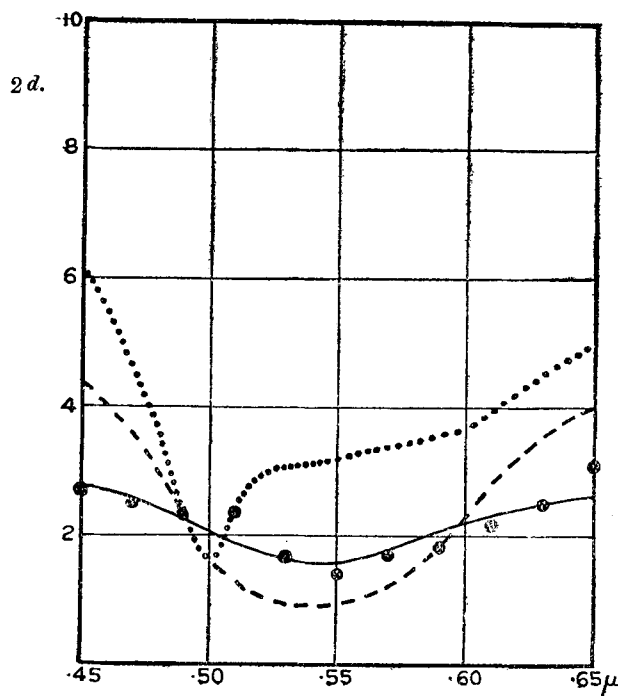
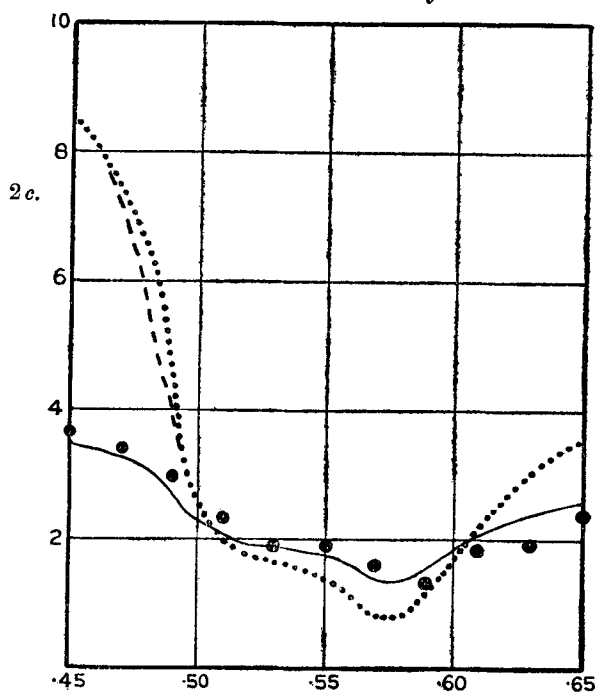
from white, yellow-white (tungsten lamp), red, and blue. The latter colours are selected, near but not on the spectrum, to correspond approximately to the coloured glasses which were used in the experimental work *. These distances

2b.



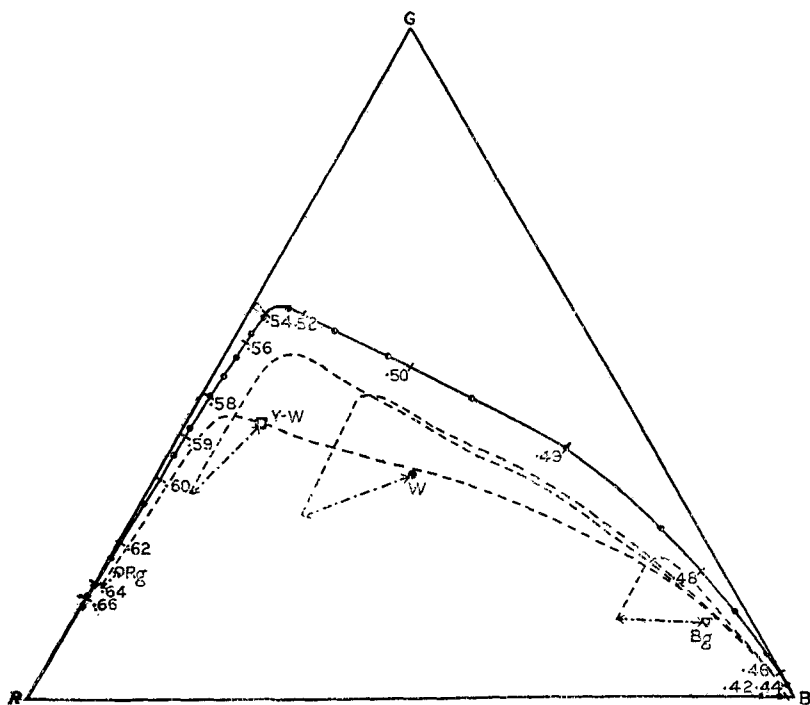
would be proportional to $\frac{2c}{\delta_H}$, if equal hue steps were represented by equal distances in the sensation triangle. Actually the latter is not the case, and it becomes necessary to plot the equal luminosity mixture point on the colour triangle, and then determine, as well as existing information will permit, the size of the just noticeable hue step at this point.

* The blue light, obtained with a blue glass and high efficiency incandescent lamp, lies much nearer the white centre than the point chosen to represent it, but the very wide slit necessarily used in this part of the spectrum, with the consequent impurity, leaves spectrum and blue comparison light probably in proper relative positions.



In fig. 3 is shown the Koenig equal sensation triangle as modified by the present writer *, in which are plotted the spectrum, the white, the yellow-white, red and blue, above mentioned, together with the lines giving the equal luminosity mixture positions of the spectrum with each of these.

Fig. 3.



Full line: spectrum in Koenig equal-sensation-sum triangle, as modified by the writer.

Heavy dots: equal hue steps in spectrum.

Rg: approximate position of red comparison light used.

Bg: approximate position of blue comparison light used.

W: white.

Y-W: Yellow-white of tungsten lamp.

Dashed lines: loci of equal luminosity mixtures of each comparison light with spectrum.

When it comes to the equal hue steps in this colour triangle all that we have is the division of the spectrum into equal hue steps by the just perceptible wave-length intervals

* See footnote, p. 103.

as determined by Steindler and others. Nutting * has constructed an equal hue difference scale of the spectrum by this means, and the black dots placed on the spectrum line represent his division. These dots show equal hue distances in the colour triangle, *for the immediate neighbourhood and in the direction that the spectrum line there runs.*

A discussion of the law by which hue changes occur in the colour triangle will not be undertaken here. Probably equal hue steps are represented by equal logarithmic increments starting from the vertices and sides of *some* triangle, apparently neither the equal sensation nor the equal luminosity one †. For the present purpose it is sufficient to note first, that these steps are smaller near the sides and vertices, and that the steps are approximately three or four times as large in going in the blue direction as in going at right angles thereto.

On the basis of this purely experimental knowledge it is possible to apply approximate corrections to the distances plotted by the dotted lines in fig. 2. Thus if we take as our unit in the spectrum-white hue step calculations the hue step where this is shortest, namely, in going from the spectrum parallel to the red-green side, near 0.5μ , then the

quotient $\frac{2c}{\delta_H}$ applying to the mixture with the yellow and

green parts of the spectrum will be subject to a reduction to as little in places as one-fourth. Such corrections, which of course can only be approximate, are shown by the dashed lines of fig. 2. They are of dominating influence in the case of the white and blue comparison lights, and of less effect with the red and yellow-white. In the case of the yellow-white the equal luminosity mixture line falls on the yellow side so much nearer the triangle side as to profit by the resultant decrease of size of the equal hue distances, with the result of keeping the distances in hue steps nearly proportional to the triangle distances.

In regard to the final step, that of finding the critical speeds from these calculations, these are to be read off directly, in arbitrary units, from the plotted curve of fig. 1, provided the value of δ_H for some definite colour triangle

* "A Method of Constructing the Natural Scale of Pure Colour," Nutting, Bulletin Bureau of Standards, vi. p. 89 (1909).

† In the first paper on flicker photometer theory it was assumed for purposes of calculation that equal steps along a mixture line are equal hue steps. No important change is made in the result there obtained by the actual apparently varying size of these steps.

distance is known. In fig. 2 speeds have been calculated on the basis of $\frac{2c}{\delta_H} \approx 500$ for the distance between red and yellow-green, these being the colours for which a value of this order of magnitude was obtained in the previous work quoted.

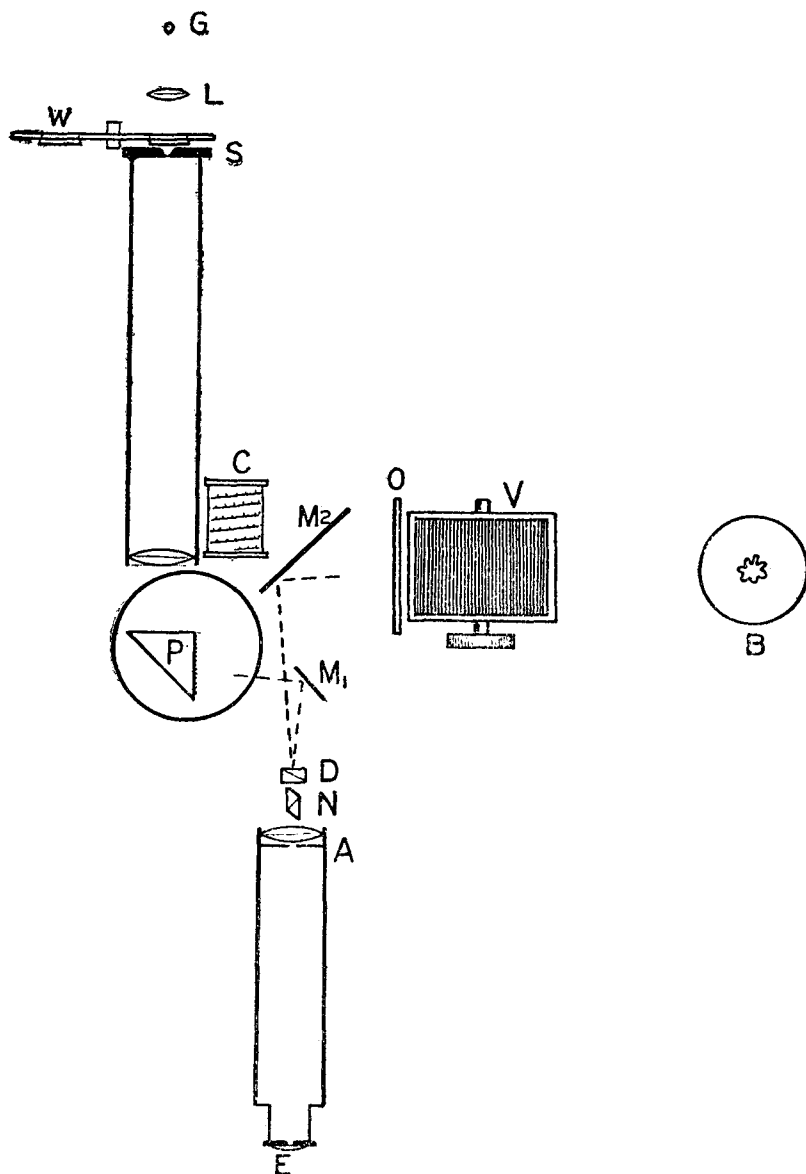
It need hardly be pointed out that the final speed calculations are based on altogether too meagre quantitative data to expect the results to be more than qualitative. They do, however, indicate clearly the nature of the wave-length critical speed curve to be expected if the underlying theory is correct. Perhaps the most important point brought out is that the curve in question *has no connexion with the luminosity curve of the equal energy spectrum*, as suggested by Troland. For while the luminosity curve retains its shape unaltered as the colour of the comparison light is varied, the critical speed curve, if the theory is borne out, can have its minimum anywhere in the spectrum. The luminosity curve which is most closely represented by the speed curve's reciprocal is that of the comparison light, but a moment's consideration of what would happen to the critical speeds against a monochromatic comparison light, which has zero value through a large part of the spectrum, disposes at once of any speculation along this line.

Experimental.

The experimental apparatus is shown in plan in fig. 4, which with its key should be clear without detailed description. Two features deserve discussion. One is the variable neutral tint screen at V, by means of which the illumination from the comparison lamp L is varied*. The procedure was to slowly change the current through the Nernst glower at the spectrometer slit, if necessary also changing the size of the slit, until the flicker setting as made by movement of the variable neutral tint screen attained a definite value. Speed settings were then made for this brightness, which was thus the same for all parts of the spectrum. The second point is the use of a set of coloured glasses, W, at the slit, each of rather narrow spectral transmission, by means of which scattered light of other wave-lengths may be practically eliminated at any point.

* Described in paper by Kingsbury, Journal Franklin Institute, August 1915, p. 219.

Fig. 4.



Plan of Apparatus.

- G—Nernst glower.
- L—Lens to form image of glower on slit.
- W—Wheel of coloured glasses to eliminate stray light.
- S—Double bilateral slit.
- C—Wave-length drum of constant deviation spectrometer.
- P—Right angle prism and transmission diffraction grating.
- M₁—Mirror reflecting spectral light.
- D—Double image prism.
- N—Rotating Nicol prism.
- A—1° diameter aperture.
- E—Observing slit, $1/2 \times 2$ mm.
- M₂—Mirror reflecting comparison light.
- O—Opal glass.
- V—Variable neutral tint screen.
- B—Comparison light.

For the red comparison light a 100 watt tungsten incandescent lamp was used together with a piece of copper ruby glass. This combination was a very close match to the spectrum at 0.63μ . For the blue a 500 watt tungsten lamp in combination with a copper and cobalt blue glass gave a fairly pure blue light. For the yellow-white was used a 1.25 w.p.c. tungsten lamp. For the white was used a 0.65 w.p.c. tungsten with "daylight glass," the latter being the glass developed in this laboratory *, and so chosen for thickness that the light transmitted from the lamp was accurately that of a black body at 5000° absolute, as shown by its match with the field of the "Apparatus for the Spectroscopic Synthesis of Colour," elsewhere described †, when operated with the appropriately calculated disk.

The experimentally found critical speeds are shown by the heavy points in figs. 2 *a*, *b*, *c*, and *d*. Owing to the fact that no effort was made to have each case studied at the same field brightness, the actual speeds in the various series have no significance, and they have accordingly been plotted in such units as will most evenly distribute their deviations from the calculated curves. The highest speeds attained were about fifteen cycles per second ‡.

* "The Development of Daylight Glass," Brady, Trans. Illuminating Engineering Society, ix. no. 9, 1914, p. 939.

† "An Apparatus for the Spectroscopic Synthesis of Colour," Ives & Brady, Journal Franklin Institute, July 1914, p. 89.

‡ No measurements have as yet been made correlating critical speeds in the polarization flicker photometer with brightness. The speeds are lower for the same brightness than in the abrupt transition flicker photometer.

Examination of these results shows that the predictions of the theory are borne out quite as well as might be expected from the nature of the data used for the calculations, remembering that the colour sensation and hue data vary from individual to individual in quite marked manner, so that exact calculations could never be made in the manner indicated without a complete study of all the colour characteristics of the eye of the particular observer used, for the field size and level of brightness chosen. The main point at issue, whether the critical speed curve is a luminosity or hue difference phenomenon, is very clearly answered by the results obtained from the red and blue comparison light sources, with their speed minima in the red and blue of the spectrum, as was to be expected. The higher speeds for the blue end as against the red end of the spectrum in using the yellow-white comparison lamp are also exactly in line with the theory here given.

Summary.

The subject-matter of this paper may be briefly summarized in the statement that a method of calculating critical speeds of disappearance of colour flicker from colour sensation and hue discrimination data has been developed and tested by experiment.

I take pleasure in acknowledging the assistance of Mr. E. R. Morton in the construction of the apparatus used, and of Mr. E. F. Kingsbury and Dr. E. Karrer in securing the readings.

The United Gas Improvement Co.,
Physical Laboratory,
Nov. 22nd, 1916.

X. *On Periodic Convection Currents in the Atmosphere.* By HAROLD JEFFREYS, M.A., M.Sc., Fellow of St. John's College, Cambridge*.

IN the mathematical treatment of the motion of the atmosphere, it is customary for meteorologists to neglect in the equation of vertical motion the terms depending on the velocity. This is implied in the hypothesis that the barometric pressure at any point is the weight of a vertical column of air of unit cross-section and infinite height, with its lower extremity at the point considered.

* Communicated by the Author.